

# Cost recovery / allocation ideas

Michael Kuczynski  
mgk1002@cam.ac.uk

## Friends plan a picnic: sharing a joint cost

- Alsatian: I can take the day off
- Carinthian: I know a really nice spot
- Friesian: I know how to light a fine fire
- Galician: I can bring a trusty corkscrew
- Illyrian: I know a great supplier of woollen rugs
- Karelian: I know where to borrow a large basket
- Ligurian: I can bring my cousin, who is so very nice

Food and drink, anyone?

[End notes are referred to in brackets]

# Preliminary: two approaches to setting prices

- Ramsey-pricing
  - Price – marginal cost =  $\lambda_R \times \{1/(\text{price-elasticity of product / activity demand})\}^{[1]}$
  - Basic idea: price is raised as price-sensitivity of user-demand drops
  - Focus is on differentiated willingness to pay
- Axiomatic-cost (Aumann-Shapley) pricing
  - Price =  $\lambda_{AS} \times \{\Sigma \text{ marginal cost added by product or activity to any cost-combination of activities}\}^{[2]}$
  - Basic idea: free-loaders (who may be price-sensitive) do have to pay something <sup>[3]</sup>
  - Focus is on differentiated contribution to fixed cost

# Basic message

- Somehow or other costs must be defrayed, paid for, covered
  - Through taxes, charges to benevolent third-parties, to other interested parties, direct beneficiaries, different types of users or of use, etc.
- How and when it is to be done raises difficult issues
  - Administrative incentives, gaming effects, rent-seeking, political viability, distributive concerns, etc.
  - Market mechanisms come with a price-tag of their own: evaluation of costs, drawing-up of contracts, etc.
- This is all side-stepped in order to concentrate on pricing options in covering cost
  - At which interface price applies is a separate matter <sup>[4]</sup>

## Nature of an ‘infrastructural’ fixed cost

- There is joint use: many planes, same runway
  - It is a ‘commons’, though not necessarily tragic [5]
- But not necessarily equal (capacity) use
  - Large 747s, small Cessnas use the same runway [6]
- Variable (generally expanding) capacity
  - Libraries get bigger, better, more inter-active [7]
- Low-to-zero additional individual user-cost
  - Within the current capacity constraint [8]
- Congestion is generally undesirable
  - At least beyond certain limits

# Three approaches to revenue generation

- By cross-subsidy
  - Take from Peter to pay for Paul: generate additional revenue by product development / enhancement, or by carrying advertising <sup>[9]</sup>
  - May be administratively efficient, yet sometimes allocatively inefficient <sup>[10]</sup>
- By a pricing or charging scheme
- By inter-temporal transfer
  - Defray the costs of a multi-period activity by spreading the burden across periods
  - Note: charging fee-for-service within a period is uncontroversially correct <sup>[11]</sup>

# What do prices do?

- Reconcile supply and demand
  - This may be claimed to maximize welfare
  - An objective function is identified as maximand
  - The task is ambitious, simple, ancient <sup>[12]</sup>
- Allocate cost
  - This typically aims at achieving administrative incentives or political, presentational requirements
  - Limited in ambition, sophisticated, modern
- Not mutually exclusive
  - Cost allocating prices *may* be demand compatible <sup>[13]</sup>

The market-clearing is uncomplicated if:

- Fixed costs are small, variable costs predominant
- Marginal cost is therefore rising
  - Additional supply is at higher unit-cost
- The market-clearing, welfare-maximizing rule

**Price = marginal cost**

therefore doesn't involve loss

- Hence the incentive and political problems of grants / subsidies don't arise
- This is however irrelevant to a joint-cost facility (except for distribution costs as discussed above)

## An aside intended mainly for economists

- It is not unusual for an infrastructural fixed cost to be heavily made up of labour cost
  - Despite the fact that labour is often thought of as a variable cost<sup>[14]</sup>
- Examples are numerous:
  - Many security systems (police, army, etc.)
  - Garbage collection
  - Throwing great parties, holding great meetings
  - Research libraries, databases
  - Steady staff numbers *vs* rising hybridizations/year<sup>[15]</sup>

## Consequent fixed-cost issue

- Declining or zero marginal costs  $\Rightarrow$  losses
  - Unless current capacity is (undesirably) strained, so that marginal costs rise
- Losses  $\Rightarrow$  potential incentive issues arising from risk of administrative slack <sup>[16]</sup>
  - Unless activity is strongly/strangely motivational
- Hence the constraint:
  - Revenue =  $k \times$  cost                      Loss =  $R - kC$
  - $k \geq 0$ ,  $k \leq p$ ,  $p \geq 1$                $p > 1 \Rightarrow$  profit <sup>[17]</sup>
- What is the appropriate scheme to generate  $k$ ?
  - How high  $k$  is or should be is not discussed here. <sup>[18]</sup>

## Standard bounded-loss, $k \leq 1$ , approaches

- Lump-sum user-payment
  - Poll tax principle, can distinguish between users, but not by extent of use [19]
  - Readily coupled with marginal-cost pricing into standard “two-part tariff”
- Demand-dependent user-payment
  - Ramsey principle, needs to distinguish users by identifying sensitivity of their demand to price
  - Widely used in several “variable load” varieties [20]
- Both justified by welfare maximization
  - Though without regard to distributive concerns [21]

## An additional standard special case

- Suppose the fixed joint-cost is in fact variable in the long run
  - Even if the library at Alexandria doesn't burn down, the Goths don't do mathematics in Greek
  - Eventually any database may become redundant
- Efficient pricing is then long run marginal cost
  - Ordinary (short run) marginal cost plus the “coupon” value of eventual replacement <sup>[22]</sup>
  - If the long-run is infinity, coupon-value is the “test discount rate”  $TDR \times \text{fixed cost to be covered}$ , which depends on the value chosen for  $k$  <sup>[23]</sup>

## Cost-axiomatic alternatives

- Consider a multi-product activity, with prices fixed by managers (or funding agencies) to allocate given costs among products,

- Not necessarily to skim off willingness to pay
- Basically cost-sharing prices, the different products engaging in a cooperative “picnic” game

- Go from Revenue =  $k \times$  cost to

$$\sum_{i=1}^n p_i q_i = k \times C(q_1, \dots, q_n)$$

- By following sensible axioms,

- So that  $p_i$  is set to be the average marginal contribution that each product makes to the product ‘coalitions’ that could possibly be formed around a (varying) fixed cost.<sup>[24]</sup>

# Aumann-Shapley value/price

$$p_{i,v}^{AS} = \frac{1}{n!} \sum_{col} m[S(col,i),i] \quad \underline{\text{AS price}}$$

is the average marginal contribution made by a product to the value of coalitions otherwise not including it

$$m[S(col,i),i] = v[S \cup \{i\}] - v[S]$$

$i \notin S$                       Coalition value including  $i$       Coalition excluding  $i$

In effect this pricing scheme is a set of arm's length, demand-independent cost-allocations by price

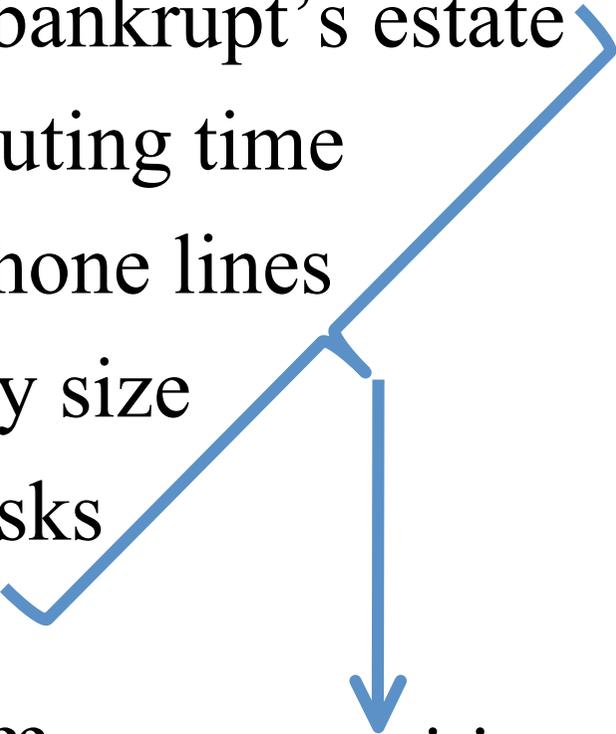
The values  $m$  depend on the axioms constraining the exercise

## Why or how axiomatic?

- Computed solution depends on axioms for ‘fair’ pricing rules:
  - Prices to be independent of units (rescaling)
  - Same price for same influence on cost (consistency)
  - With sub-costs additive, price is additive (additivity)
  - If costs are positive, prices cannot be negative (positivity)
  - Revenue constraint is met  $p(C, \mathbf{q}) = k \times C(\mathbf{q})$
- Fairness refers to joint-cost burden sharing
  - Not to inter-user equity, progressive income distribution, etc.<sup>[25]</sup>

# Axiomatic-cost examples

- Allocation of claims to a bankrupt's estate
- Allocation of super-computing time
- Access to dedicated telephone lines
- Airport runway charges by size
- Allocation of insurance risks



Users require different capacities

different product varieties

fair cost-allocating charge

## A different more modest proposal

- Infrastructure, even if expanding, only pays off over time
  - Present users are not the only, nor perhaps even the main beneficiaries of an expanding database
- This suggests cost-recovery over time is appropriate
  - Future taxpayers to pay as well as, or more than, current
- The appropriate cost-recovery vehicle is therefore a form of *bond* finance
  - A perpetuity whose coupon is  $\text{TDR} \times k \times \text{cost of facility}$

Recall: there have been big practical omissions

- Mechanics of the conduits for cost-recovery
  - How much should perhaps be paid: how high  $k$ ?
  - Who exactly pays whom (e.g., does grant go to infrastructure user who makes the payment, or direct to infrastructure provider, as in the open-access case)?
- Costliness of market mechanisms
  - Using prices isn't costless, to buyer or to seller
  - There are information and contracting costs, verification costs, insurance, etc.
- This has been Hamlet, *not* the Prince of Denmark
  - How to charge, but not whom to charge

# End notes

1. This result is derived by maximizing a general objective function (some measure of ‘welfare’) subject to an equality budget constraint that revenue equal  $= k \times \text{cost}$ , where revenue is  $p_i \times q_i$  ( $p_i$ ) and the price-elasticity of  $i$ -th user’s demand is  $\partial q_i / q_i / \partial p_i / p_i$ .  $\lambda_R$  depends on  $k$  as defined below
2. This result is derived by considering a cooperative interaction between  $n$  different activities sharing a joint cost, such that each pays an average of the marginal contribution it would make to the joint cost by joining in any of the permutations of potential cost-sharing activities, so  $\lambda_{AS} = 1/n!$
3. If a large truck transports my small bag together with all your possessions, my marginal cost seems zero; yet without the truck I would have a transport-cost to bear.
4. Prices (e.g., Ramsey or Aumann-Shapley) may be levied by facility-operator on final user, or by final user on a funding intermediary, &c.
5. In the ‘tragedy of the commons’ weakly-established rights-recognition leads to over-use or under-provision. But r-recognition can be improved or accepted.
6. This parallels the slide shown by Martin Hrabé: at Munich airport a small plane seems to imply only a small marginal cost, once the airport has been built to accommodate the large; yet the small still needs a runway.
7. Databases develop in quality thru’ use as well as thru’ addition.
8. Given current quality, marginal cost  $\approx$  zero (distribution costs aside)

## End notes continued

9. Museums sell postcards, etc.
10. Cutting through a thicket of complications, cross-subsidy is unacceptable if it serves to suppress competition ('predatory' pricing) and acceptable if it allows provision of a socially useful service in an otherwise non-profitable market.
11. On the administratively practical principle that "each tub should stand on its own bottom", there is no reason for the cost of the individual use of a joint-cost facility (such as packaging and distributing specimens) not to be charged as fee-for-service: a genuine (short run) marginal cost.
12. This is the focus of the "invisible hand" approach to the role of prices.
13. In particular Aumann-Shapley cost-sharing prices can be constructed to be demand-consistent.
14. In fact, because of social charges and other features of employment, in practice labour approximates a fixed cost, independent of activity-output.
15. This refers to the evidence presented on ArrayExpress.
16. If tubs don't "stand on their own bottom", a loss that might be justifiable in terms of marginal-cost pricing may conceal instead a loss due to administrative slack.
17. Ordinarily a joint-cost facility is required, on administrative grounds, to show bounded-loss rather than actual profit, i.e.,  $k \leq p$  with  $p \leq 1$ .
18. For reasons aired above under the heading "message".
19. This sort of subscription-payment can raise issues of plain equity.

# End notes concluded

20. Ramsey pricing is widely used in utilities, e.g., when peak-time use is charged at a higher rate.
21. Both lump-sum subscription and Ramsey-prices for use are allocatively efficient: customers pay according to willingness to pay for the unit of service. But they can result in undesirably high charges for users whose use is socially desirable (e.g., impecunious research students).
22. The coupon value of eventual replacement refers to what needs to be set aside in each period, in a sinking fund, in order to have at hand the capital sum needed when replacement is due.
23. If the library or database lasts forever, it is assumed replaced at the end of time. TDR is the rate at which an intelligent taxpayer, *contd* disembodied from the consequences of his vote for his own purse, would discount future benefits relative to current. It is a controversial matter, as discussion of the Stern [2007] review of climate-change measures shows.
24. To determine each user's charge, shuffle each user into all the possible coalitions (factorial  $n$ ) it could form with the other  $n - 1$  users, then take the average of the marginal contribution that its presence in each coalition makes to the joint-cost. See next slide. The average depends on  $k$ .
25. In the airport example, the big planes may carry budget passengers and the small ones executives: the AS value doesn't take this into account, only that large planes need more runway.